Differentiation

Differentiation Rules

1. For any x^b, its derivative is simply $b \times x^{b-1}$.

The derivative of x^5 is $5x^4$.

2. For any ax^b its derivative is $b \times ax^{b-1}$. The power is put out in front of the expression, and then the power is reduced by 1. Remember anything to the power 1 is just itself, and anything to the power 0 is 1.

The derivative of $3x^4$ is $12x^3$.

The derivative of $3x^{-4}$ is $-12x^{-5}$

3. When you have terms separated by '+'s or '–'s you can find the derivative of each term then add them together to find the derivative of the whole function.

The derivative of $2x^4 + 3x^2 - 3x + 2$ is $8x^3 + 6x - 3$

Note that the 2 just disappears. All constants (numbers by themselves without algebraic symbols) disappear when you find the derivative.

4. The derivative of sin x is cos x

5. The derivative of cos x is –sin x (note the change in sign).

6. The derivative of e^x is e^x.

7. The derivative of ln x is 1/x, for x > 0. There is no ln x when x ≤ 0 (ln is the natural logarithm)

Note that the derivative can be written a few different ways:

- If you have $f(x) = \ldots$, then the derivative can be written $f'(x) = \ldots$
- If you have $y = \ldots$, then the derivative can be written $y' = \ldots$
- If you have a function with 'x' as the variable being differentiated, the derivative can be written as:

$$\frac{d}{dx}(something)$$

'Something' is the function which has 'x' in it.

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

In words, this says the derivative of y with respect to x is equal to the derivative of y with respect to u times the derivative of u with respect to x. Note if you cancel out the 'du's on the right hand side, you are left with the left hand side.

$$4(3x+4)^5$$

$$\frac{dy}{du} = 20u^4 \text{ and } \frac{du}{dx} = 3$$

So we can use our differentiation rules to calculate that

$$\frac{dy}{dx} = 20u^4 \times 3 = 60u^4.$$

We can then substitute in for what u is and give the answer as

$$60(3x + 4)^4$$

Product Rule

$$y = (3x^2 + 2x)(2x^4 - 5)$$

The product rule simply states that if the function can be split up into a sub-function multiplied by another sub-function, then:

$$y' = u'v + uv' \quad \text{or} \quad \frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$

Let u = $3x^2 + 2x$ which means that $u' = 6x + 2$

Let v = $2x^4 - 5$ which means that $v' = 8x^3$

$$\frac{dy}{dx} = (6x+2)(2x^4 - 5) + (3x^2 + 2x)8x^3$$

$$= 36x^5 + 20x^4 - 30x - 10$$