More Differentiation

Quotient Rule

This rule is for when you can split your function into one sub-function divided by another sub-function. If the two sub- functions are u and v:

$$y = \frac{u}{v}$$

Then:

$$y' = \frac{u'v - uv'}{v^2}$$

Example:

Find the derivative of $y = \frac{5x}{(6x^2 + 2x)}$

Firstly:

Let u = 5x

Let v = $6x^2 + 2x$.

Which gives:

$$u' = 5$$

$$v' = 12x + 2$$

Then:

$$y' = \frac{5 \times (6x^2 + 2x) - 5x \times (12x + 2)}{(6x^2 + 2x)^2}$$

$$= \frac{30x^2 + 10x - 60x^2 - 10x}{(6x^2 + 2x)(6x^2 + 2x)}$$

$$= \frac{-30x^2}{x^2(6x + 2)(6x + 2)}$$

$$= \frac{-30}{(6x + 2)^2}$$

Handy Hint - Writing letters in calculus

Be careful when you write 'u' and 'v' – it is easy to mix them up! You could use the letters 'a' and 'b' just as easily, 'u' and 'v' are used here because many textbooks use them.

Handy Hint - Starting the question

When you get a derivative question, your first decision should be which rule to use. You can then write your 'u's etc. out so they are easy to use:

$$u = \ldots$$

$$u' = \ldots$$

$$v = \ldots$$

$$v' = \ldots$$

Handy Hint - Product or quotient rule

Any quotient rule problem can be turned into a product rule problem. For example:

$$y = \frac{5x}{(6x^2 + 2x)}$$

$$= (5x) \times (6x^2 + 2x)^{-1}$$

$(6x^2 + 2x)$ is really $(6x^2 + 2x)^1$. When we change it from dividing to multiplying, we change its power from 1 to –1. In effect we are moving it from the denominator to the numerator – the sign of the power always changes when we do this.