Adding indices when you multiply

When you multiply together two identical numbers or pronumerals that are both raised to a power, you can combine them together by just *adding* the indices together:

$$x^a \times x^b = x^{a+b}$$

$$4^2 \times 4^3 = 4^{2+3} = 4^5$$

Subtracting indices when you divide

When you divide identical numbers or pronumerals each raised to a power, you need to *subtract* the indices from each other instead of adding them together.

$$x^a \div x^b = x^{a-b}$$

$$2^5 \div 2^3 = 2^{5-3} = 2^2$$

Raising powers to powers

When you raise a power to *another* power, you can rewrite the whole expression, combining the two indices or powers by *multiplying* them together:

$$\left(x^a\right)^b = x^{a\times b}$$

$$\left(4^2\right)^3 = \left(4^{2\times 3}\right) = 4^6$$

The zero index or exponent

When you raise *anything* to the power zero, you end up with the number 1:

$$\text{Anything}^0 = 1$$

Negative indices or exponents

When you raise a number to a negative power, it's equivalent to the *inverse* of that number raised to the positive of that power:

Index and Log Laws

$$x^{-a} = \frac{1}{x^a} \qquad 2^{-1} = \frac{1}{2}$$

Indices and exponents that are fractions

The general way to convert from a fractional power to a root is like this:

$$a^{1/x} = \sqrt[x]{a} \qquad 5^{1/2} = \sqrt{5} \qquad 5^{1/4} = \sqrt[4]{5}$$

Fractional indices with numerators larger than one

$$2^{2/3} = 2^{1/3+1/3}$$

$$2^{2/3} = 2^{1/3} \times 2^{1/3}$$

$$2^{2/3} = \sqrt[3]{2} \times \sqrt[3]{2}$$

$$2^{2/3} = \left(\sqrt[3]{2}\right)^2$$

Common cube roots

$$\sqrt[3]{1} = 1,\ \sqrt[3]{8} = 2,\ \sqrt[3]{27} = 3,\ \sqrt[3]{64} = 4,\ \sqrt[3]{125} = 5$$

Log Laws

If we have:

$$a^c = b$$

The log statement is:

$$\log_a b = c$$

This statement says that the *base a* to the *power c* equals *b*.

Common logarithms

These all have a base of 10. For instance:

$$\log_{10} 10 = 1,\ \log_{10} 100 = 2,\ \log_{10} 1000 = 3$$

Natural logarithms

These all have the special number 'e' as a base, which is equal to about 2.718. Natural logarithms are also expressed using ln instead of log.

$$\ln_e e = 1 \qquad \ln_e e^2 = 2$$

Logarithm Laws

1. **$\log_A BC = \log_A B + \log_A C$**

   $\log_2 6 = \log_2 3 + \log_2 2$ since $2 \times 3 = 6$

2. **$\log_A (B/C) = \log_A B - \log_A C$**

   $\log_3(3) = \log_3 6 - \log_3 2$ since $6/2 = 3$

3. **$\log_A (B)^D = D \times \log_A (B)$**

   $\log_4 16 = \log_4 4^2 = 2 \times \log_4(4)$ since $16 = 4^2$

4. **$\log_{\text{some number}}$ (same number) = 1**

   $\log_{57} 57 = 1$

Change of Base Theorem

$$\log_B C = \frac{\log_A C}{\log_A B}$$

Example: To calculate what $\log_2 17$ is:

$$B = 2 \qquad C = 17$$

$A = 10$ since calculators have base 10 on them:

$$\log_2 17 = \frac{\log_{10} 17}{\log_{10} 2}$$

$$\approx 4.09$$

Answer check: 17 is only a bit more than 16, which we know is 2 to the power 4. An answer of 4.09 makes sense.