Integration

Integration can be thought of as the reverse of differentiation. It is like doing differentiation in reverse.

The constant of integration

Whenever you integrate something, you always have something called the constant of integration in your answer. This is a letter, usually a 'c', that represents a number.

Integration Rules

1. If I integrate ax^n, I get $\frac{a}{n+1}x^{n+1}+c$

For example:

Function	Integrated
$4x^3$	$x^4 + c$
5	$5x + c$
$-3x^{-2}$	$3x^{-1} + c$

2. Rule 1 holds for just about everything except when you have something to the power '–1', i.e. 'x^{-1}', or '$1/x$' (these are the same thing). In a special case like this, the integral of '$1/x$' is '$\ln x + c$'. '$\ln$' is the natural log.

3. The integral of e^x is $e^x + c$.

4. Integration can be done in bits. Say I have $6x^2 + 5x^4$ and I want to integrate it. I can integrate each term separately, then add the results:

First integrate $6x^2$ to get $2x^3 + a$.

Then integrate $5x^4$ to get $x^5 + b$.

Note I have used a different letter to 'c' for the *constant of integration*. Also note that I have used a different letter in each one – they do not necessarily have the same constant of integration value.

Add the results together:

$$\begin{aligned} &=> 2x^3 + a + x^5 + b \\ &= x^5 + 2x^3 + (a+b) \\ &= x^5 + 2x^3 + c \end{aligned}$$

The 'c' in the final answer represents 'a + b'. Since 'a' is just a number, and 'b' is just another number, when they are added together they just represent yet another number. To make it easier to read, we can just call this number 'c'.

5. The integral of $\cos x$ is $\sin x + c$.

6. The integral of $\sin x$ is $-\cos x + c$ (note the negative sign).

7. If you have a linear function raised to a power, you can integrate it. By linear function, I mean that there are not any squared terms or higher powers in it.

For instance:

Integrate $(3x-4)^3$

What you can do is represent the function in the brackets by a letter, say the letter 'a'. We can then rewrite what we have to integrate as a^3.

Integrate the letter normally – it becomes $\frac{1}{4}a^4 + b$.

'b' is the constant of integration. Then divide it by the derivative of the function a with respect to x. *The derivative of function 'a' with respect to 'x' is calculated like this:*

$$a = 3x - 4$$

$$\frac{da}{dx} = 3$$

So if we divide our integral by '3' we get:

$$\begin{aligned} &=> \frac{\frac{1}{4}a^4 + b}{3} \\ &= \frac{1}{12}a^4 + \frac{b}{3} \\ &= \frac{1}{12}a^4 + c \end{aligned}$$

Since 'b' is just a number, we can rewrite '$\frac{b}{3}$' as 'c' to make the equation easier to read.

The last step is to substitute in what 'a' represents:

$$\frac{1}{12}(3x-4)^4 + c$$

Integration Symbol

There is a mathematical symbol that stands for the verb integrate. Say I want to integrate '2x + 3'. I can rewrite this without using words as:

$$\int (2x+3)dx$$

The integrate symbol is '$\int$'. I also need to say what variable I am integrating *with respect to*. This is what the 'dx' does – 'dx' means the change in 'x'. It tells me that the variable I'm integrating is the one which is changing – 'x'.