Definite Integrals

$$\int (2x+3)dx$$

The mathematical expression above is called an indefinite integral. There is also a definite integral. The *definite integral* is like an *indefinite integral* except two numbers are specified between which to integrate. An example of a *definite integral* is:

$$\int_1^4 (2x+3)dx$$

The two numbers specified are 1 and 4. To evaluate what this definite integral is equal to, you do the following:

Step 1 *– Just integrate the function we have been doing – we get the answer* x^2+3x+c *.*

Step 2 *– Evaluate what this answer equals when we substitute the top number in (the 4):*

$$\Rightarrow 4^2+(3\times 4)+c$$
$$=16+12+c$$
$$=28+c$$

Step 3 – *Evaluate what this answer equals when we substitute the bottom number in (1):*

$$\Rightarrow 1^2+(3\times 1)+c$$
$$=1+3+c$$
$$=4+c$$

Step 4 *– Subtract step 3 from step 2:*

$$\Rightarrow (28+c)-(4+c)$$
$$=24+c-c$$
$$=24$$

Note how the constant of integration cancels out.

More Integration

Some Physical Meanings of Integration

There are three things that can describe a body's position and movement; position, velocity and acceleration.

- Position is where the body is located.
- Velocity is how fast the body is moving, and in which direction.
- Acceleration is a measure of whether the body's velocity is getting faster or slower, and how quickly (this is a simplified definition).

If you know one of these three things, you can obtain information about the other two using differentiation and integration.

- You can get velocity by *differentiating* the body's position function.
- You can get acceleration by *differentiating* the body's velocity function.

Likewise, you can use integration to go in reverse:

- You can get velocity by *integrating* the body's acceleration function.
- You can get position by *integrating* the body's velocity function.

Another way of visualising this is:

Differentiation: position $\longrightarrow$ velocity $\longrightarrow$ acceleration

Integration: acceleration $\longrightarrow$ velocity $\longrightarrow$ position

Example:

A truck's velocity is (5t) m/s, where t is the time in seconds since it started moving. After 10 seconds, how far has it moved from where it started?

We're interested in how far the truck has moved between the start time and the end time. So the numbers to put in are the time when the truck starts moving, and the time at the end. It starts at time = 0 and finishes at time = 10. Now we can write the definite integral:

$$\int_0^{10} (5t)dt$$

Note how now it is a 'dt' at the end instead of a 'dx'. In general, you write 'd_' and put whatever in the function is changing – in this case 't', in previous cases it was 'x'.

Integrating we get:

$$2.5t^2+c$$

We now have an expression for the truck's position after 't' seconds.

So now we work out what this expression equals for 10 seconds (the **top** number):

$$\Rightarrow (2.5\times 10^2)+c$$
$$=250+c$$

Then we work out what this expression equals for 0 seconds (the **bottom** number):

$$(2.5\times 0^2)+c=c$$

Then we subtract the second result from the first:

$$(250+c)-c=250m$$