Logic

The basic part of logic in mathematics is quite simple. It is all built around simple *binary* statements or *propositions*. These statements are ones that can either be true *or* false. Here's an example of a proposition:

My name is Michael

This statement can either be true or false. My name can't 'sort of' be Michael, it either is or it is not. Here's an example of a statement which *isn't* a binary statement:

How many names do I have?

This statement is a question. It can't be true or false, it needs a *quantitative* answer (i.e. the number of names I have).

Logic Connectors

There are five major *connectors* that you can use in logic to write *logic sentences*.

OR connector

The $\vee$ symbol is the 'OR' symbol – it means at least *one* of the two things on either side of it will happen. Remember the 'or both' part – it's commonly forgotten.

This afternoon I will play tennis OR I will read (or both)

If I used 't' to describe tennis, and 'r' to describe reading, then I could rewrite this statement:

$$t \vee r$$

AND connector

This afternoon I will play tennis AND I will read

The mathematical symbol for 'AND' is $\wedge$:

$$t \wedge r$$

IMPLIES connector

Jason goes to primary school – this implies he is less than 15 years old

It's a bit trickier to analyse whether an imply statement is true or false. There are three possibilities:

- The first part of the statement is true, and the second part is true as well. In that case the *implication* is *true*.
- The first part of the statement is true, but the second part is false. In that case the implication is *false*.
- The first part of the statement is false. In this case, the implication is *always* true. This is sometimes confusing for people. Whether the second part of the statement is true can only be judged when the first part is true – the second part *leads on* from the first part. So the convention is that when the first part is false, we just say that the whole meaningless implication is *true*.

The mathematical symbol for IMPLIES is an arrow with two lines – $\Rightarrow$. If we use 'PS' for primary school and 'K' for kid under 15 years old, the mathematical way to write the last statement is:

$$PS \Rightarrow K$$

EQUIVALENCE connector

I will play tennis this afternoon if I read a book, and I'll read a book if I play tennis.

The mathematical symbol for EQUIVALENCE is a double headed arrow – $\Leftrightarrow$.

$$t \Leftrightarrow r$$

NOT connector

The logic symbol for this NOT connector is '~'. So if I start with a statement like:

I will play tennis this afternoon

I can convert this into a mathematical statement just like this:

$$t$$

If I put a NOT symbol in front of this, I reverse the statement:

$$\sim t$$

This reverses the statement so it is now:

I will not play tennis this afternoon

Truth Tables

Truth tables are a way of writing down all the possible combinations of statements and saying whether the whole combined statement is true or false.

OR		
A	**B**	$A \vee B$
True	True	True
True	False	True
False	True	True
False	False	False