Matrices

What is a Matrix

A simple matrix is a two-dimensional clump of numbers. It has a length and a height. For instance, this is a *two by three* matrix:

$$\begin{bmatrix} 4 & 17 & -5 \\ 22 & -12 & 7 \end{bmatrix}$$

The first dimension mentioned is the number of rows – two. The second dimension mentioned is the number of columns – three.

The order of a matrix

You always describe the number of *rows* first, then the number of *columns*. This matrix has two rows and three columns, which is why it's described as a two by three matrix. This is often called the *order* of the matrix – the number of rows by the number of columns.

Two rows

Three columns

$$\begin{bmatrix} 4 & 17 & -5 \\ 22 & -12 & 7 \end{bmatrix} \qquad \begin{bmatrix} 4 & 17 & -5 \\ 22 & -12 & 7 \end{bmatrix}$$

In a two by three matrix, there are six *elements* – cells or places where a value is.

Describing an element

The 22 could be described in this way:

$$A_{21} = 22$$

This says that the value in row 2, column 1 of matrix A is equal to 22. To talk about the '–5' element we'd say:

$$A_{13} = -5$$

Adding and subtracting matrices

You can only add or subtract one matrix with/from another if they have the same order – the same number of rows *and* the same number of columns. To actually do the calculation, all you do is perform the addition or subtraction on each pair of *corresponding* elements (the elements in the same position as each other):

$$\begin{bmatrix} 2 & 7 \\ -15 & 4 \end{bmatrix} + \begin{bmatrix} -5 & 14 \\ -7 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 2+-5 & 7+14 \\ -15+-7 & 4+-8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 21 \\ -22 & -4 \end{bmatrix}$$

Multiplying matrices

To do multiplication, you start at the *top row* of the first matrix, and multiply each of these elements by the ones down the *first column* of the second matrix.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times 7 + 2\times 9 & ? \\ ? & ? \\ ? & ? \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times 7 + 2\times 9 & 1\times 8 + 2\times 10 \\ 3\times 7 + 4\times 9 & ? \\ ? & ? \end{bmatrix}$$

The number of *rows* in your new matrix will be equal to the number of rows in your *first* matrix. The number of *columns* in your new matrix will be equal to the number of columns in your *second* matrix. So its order will be 3×2.

The next step in the calculation is to stick with the same row in the first matrix, but multiply it by the elements in the *second* column of the second matrix, like this:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times 7 + 2\times 9 & 1\times 8 + 2\times 10 \\ ? & ? \\ ? & ? \end{bmatrix}$$

Now we've finished with the first row in both our first matrix *and* our answer matrix. We move on to the second row of our first matrix. The first step is to multiply it by the first *column* of the second matrix:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times 7 + 2\times 9 & 1\times 8 + 2\times 10 \\ 3\times 7 + 4\times 9 & ? \\ ? & ? \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times 7 + 2\times 9 & 1\times 8 + 2\times 10 \\ 3\times 7 + 4\times 9 & 3\times 8 + 4\times 10 \\ 5\times 7 + 6\times 9 & 5\times 8 + 6\times 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 25 & 28 \\ 57 & 64 \\ 89 & 100 \end{bmatrix}$$

Multiplying and dividing by a scalar

A scalar is a single number, rather than a group of numbers like in a matrix. All you do is perform the multiplication / division individually on each element in the matrix.

$$\begin{bmatrix} 14 & 19 \\ 7 & 6 \end{bmatrix} \times 4 == \begin{bmatrix} 14\times 4 & 19\times 4 \\ 7\times 4 & 6\times 4 \end{bmatrix} = \begin{bmatrix} 56 & 76 \\ 28 & 24 \end{bmatrix}$$