More Matrices

Identity Matrix

The term 'identity matrix' is used to describe any *square* matrix (same number of rows as columns) which has ones down its primary diagonal, and zeros everywhere else. The primary diagonal starts at the top left corner of the matrix and continues down to the bottom right corner. For instance, these are both identity matrices:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The *subscript* of an I (a little number to the right and below the 'I') is used to describe how large the identity matrix is.

Any matrix you multiply by an *identity* matrix, you just get the original matrix again as your answer.

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+2\times0 & 1\times0+2\times1 \\ 3\times1+4\times0 & 3\times0+4\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The Inverse of a Matrix

$$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

The inverse of this matrix is the matrix you need to multiply it by to get the identity matrix.

Finding the matrix inverse

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

We can find the inverse like this:

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The 'ad – bc' part in the denominator of the fraction is called the *determinant* of the matrix A, and is often written as $\det(A)$ or $|A|$:

$$A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det(A) = ad - bc$$

Work out determinant for the original A matrix:

$$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

$$\det(A) = ad - bc$$

$$a = 3 \quad b = 5 \quad c = 2 \quad d = 4$$

$$\det(A) = 3\times4 - 5\times2$$

$$\det(A) = 2$$

Swap around and change signs of the elements in matrix A:

Sign change

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$$

Sign change

Inverse of A is:

$$A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}^{-1} = \frac{1}{2}\times\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & -2.5 \\ -1 & 1.5 \end{bmatrix}$$

Check whether when we multiply A by it we really do get the identity matrix:

$$\Rightarrow A \times A^{-1}$$

$$= \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & -2.5 \\ -1 & 1.5 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times2+5\times-1 & 3\times-2.5+5\times1.5 \\ 2\times2+4\times-1 & 2\times-2.5+4\times1.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equations into Matrices

$$\begin{matrix} 3x+5y=31 \\ 2x+4y=24 \end{matrix} \quad \text{becomes} \quad \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 31 \\ 24 \end{bmatrix}$$

Multiplying both sides by the inverse of the first matrix gives you a very useful result.

$$\begin{bmatrix} 2 & -2.5 \\ -1 & 1.5 \end{bmatrix}\times\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -2.5 \\ -1 & 1.5 \end{bmatrix}\times\begin{bmatrix} 31 \\ 24 \end{bmatrix}$$

$$\begin{bmatrix} 2\times3-2.5\times2 & 2\times5-2.5\times4 \\ -1\times3+1.5\times2 & -1\times5+1.5\times4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2\times31-2.5\times24 \\ -1\times31+1.5\times24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad \text{so} \quad \begin{matrix} x = 2 \\ y = 5 \end{matrix}$$