

A proportional or direct relationship is one where the value of one variable is directly related to the value of another. For instance, a very simple direct relationship is:

$$y = 2x$$

Constant or Proportionality

In order for a relationship to be proportional, you need to be able to write it in this general form:

$$y = kx$$

‘k’ is called the *constant of proportionality*, and can be any value you need apart from zero.

You can also have direct relationships where one variable varies as the *square* of the other variable, like this one for instance:

$$y = kx^2$$

In this case we’d say that y varies directly as x².

The Proportional Sign

When one variable is directly related to another, you can use the *proportionality symbol* to describe the relationship. The symbol is a ‘∝’. So if we write something like...

$$a \propto b$$

...what we mean is that the value of ‘a’ is proportional to the value of ‘b’, and that we can write a *quantitative* (one involving actual numbers and values) relationship using just a *constant of proportionality*, like this:

$$a = kb$$

Same goes for ‘proportional to the square’ relationships, for instance for $y = kx^2$:

$$y \propto x^2$$

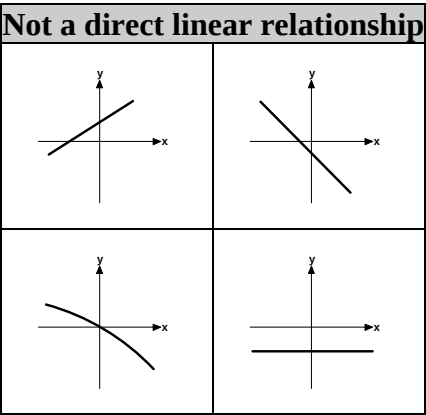
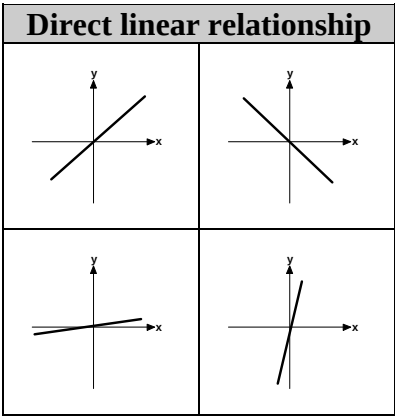
Proportional or direct relationships

Direct Linear Variation

When one variable is proportional to some constant times the other variable, this is known as direct *linear* variation.

Graphs of a direct linear relationship have these characteristics:

- The line is a straight one
- This straight line always passes through the *origin* of the graph (the 0, 0 point where the axes usually intersect)



From an Equation

Get the equation into a form where there’s one variable on each side of the equals sign. Then, if both variables are raised to the power ‘1’, and you’ve got no other terms, then you’ve got yourself a linear relationship:

Direct linear relationship	Not a direct linear relationship
$a = 5b$ $t = 7.5s$ $u = \frac{c}{23}$	$x = \frac{7}{y}$ $b = a^2$ $e = \sqrt{f}$

From a Table

It’s pretty easy to tell if there is a direct linear relationship in a table of data. Just assume that there is one, and work out the value of the proportionality constant from that. Then use the formula to check the other values in the data set.

x	2	3	4	5
y	5	7.5	10	12

$$y = kx$$

$$5 = k \times 2$$

$$k = 5 \div 2$$

$$k = 2.5$$

so $y = 2.5x$

Then use this equation to check the other values in the table. For x = 3, 4, 5:

$$y = 2.5 \times 3$$

$$y = 2.5 \times 4$$

$$y = 2.5 \times 5$$

$$y = 7.5$$

$$y = 10$$

$$y = 12.5$$

12.5 does *not agree* with the y value of ‘12’ in the table - the data in the table is not part of a direct linear relationship.