

Quadratic Equations

Quadratic equations are second degree equations which always have a variable raised to the power ‘2’.

$x^2 = 4$ is a quadratic equation, because ‘x’ is a variable and it’s raised to the power ‘2’. It could also be written $x^2 - 4 = 0$.

$x^2 + 2x - 5.5 = 0$ is also a quadratic equation, because ‘x’ is a variable, and it’s raised to the power ‘2’. This equation however also has a term where ‘x’ is raised to the power ‘1’. It’s still a quadratic equation though.

$x^3 + 2x^2 - 5 = 0$ is not a quadratic equation, because it has a variable raised to a power higher than ‘2’.

General Form

The *general* form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

The letters a, b, and c are just simply numbers. They can be fractions or decimals, negative or positive numbers. Any equation that fits this general form **or** can be rearranged to fit this general form is a quadratic equation.

For instance, the following is a quadratic equation:

$$x^2 = 4$$

It doesn’t look the same simply because this equation has ‘b’ being 0. It is really:

$$(1 \times x^2) + (0 \times x) - 4 = 0$$

Quadratic Equations

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use it to solve: $x^2 + 3x + 2 = 0$

For this equation, a = 1, b = 3 and c = 2. So:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{1}}{2}$$

$$x = \frac{-3 + \sqrt{1}}{2}, \frac{-3 - \sqrt{1}}{2}$$

$$x = -1, -2$$

Note how a ‘±’ sign means that there are actually two solutions, one for when it is a ‘+’, and one for when it is a ‘-’.

Solving By Factorising

$$2x^2 + 6x + 4 = 0$$

a = 2, b = 6 and c = 4. Think of ways to get $2x^2$:

$$(2x + \dots)(x + \dots)$$

Think of two numbers that multiply together to give c (which is 4):

$$2 \text{ and } 2, -2 \text{ and } -2, 1 \text{ and } 4, -1 \text{ and } -4$$

Let’s try 2 and 2:

$$(2x + 2)(x + 2)$$

To check whether you have a correct guess, multiply the brackets out:

$$(2x + 2)(x + 2)$$

$$= 2x^2 + 4x + 2x + 4$$

$$= 2x^2 + 6x + 4$$

If not, try different combinations of numbers.

Picking The Correct Solution

Now say you’ve just solved a quadratic equation and you’ve got two answers, t = 5, -1. For this particular problem pretend that the solutions represent how long in seconds it takes a rocket to reach a certain height. The solution t = 5 is fine, but what does the solution t = -1 mean? This suggests that the rocket reaches a certain height before it is launched!

Obviously this cannot be true, so only the solution t = 5 should be used. This can happen quite often – you must pick the ‘sensible’ solutions. (t = -1 is a mathematical solution that doesn’t make any sense when you use it in a real life application).

Only one answer to the square root of a number by itself

There is only one answer when you take the square root of a number just by itself:

$$\sqrt{4} = 2 \text{ only}$$

You only get the two ‘plus’ and ‘minus’ answers when you’ve got the square of a variable involved, and you need to take the square root of both sides, like this:

$$x^2 = 4$$

$$x = \pm \sqrt{4}$$

$$x = \pm 2$$