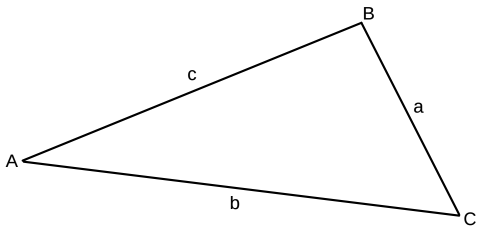
The Cosine Rule

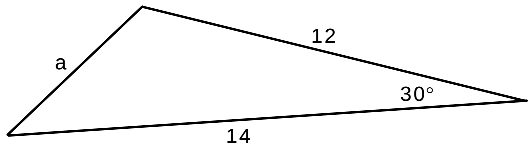


The cosine rule says that:

$$a^2 = b^2 + c^2 - 2 \times b \times c \times \cos A$$

In words, this means that, "*a* squared is equal to the sum of the squares of the other two sides in the triangle, minus two times the *product* of the other two sides times cos of the angle opposite *a*."

The cosine rule **does not need a right angle** in the triangle to work.



$$a^2 = b^2 + c^2 - 2 \times b \times c \times \cos A$$

$$a^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \times \cos 30°$$

$$a = \pm\sqrt{49.02}$$

$$a = \text{(about) } 7 \textit{ units}$$

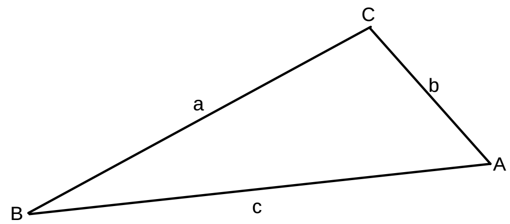
Don't forget to take the square root! Note that I have only taken the positive answer, as a negative side length does not make sense in this situation.

In general the cos rule is used to find the length of one side, when you know:

- the length of the two other sides in the triangle, and
- the angle between these two other sides.

Cosine and Sine Rule

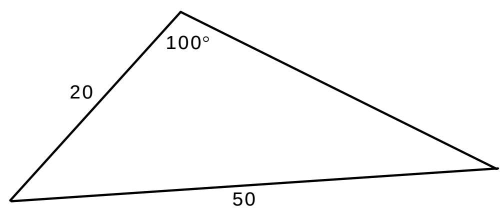
The Sine Rule



The sine rule states that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The triangle **does not** have to have a right angle in it for this rule to work.



Let a = the side of length 50

Let A = the angle opposite 'a' (100°)

Let b = the side of length 20

Let B = the angle opposite 'b'

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{50}{\sin 100°} = \frac{20}{\sin B}$$

Invert both sides

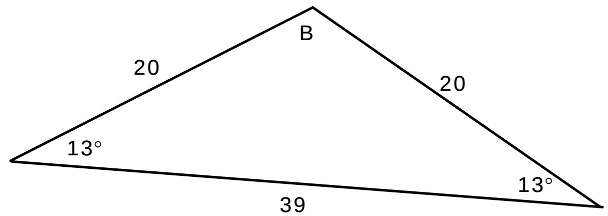
$$\frac{\sin 100°}{50} = \frac{\sin B}{20}$$

$$20 \times \frac{(\sin 100°)}{50} = \sin B$$

$$\sin B = 0.3939$$

$$B \approx 23.2°$$

Problems with the Sine Rule



If you used the sine rule to solve for the unknown angle in this triangle, you'd get:

$$\frac{20}{\sin 13°} = \frac{39}{\sin B}$$

$$\frac{\sin 13°}{20} = \frac{\sin B}{39}$$

$$\frac{39 \times (\sin 13°)}{20} = \sin B$$

$$B = \sin^{-1}\left[\frac{39 \times \sin 13°}{20}\right]$$

$$B = 26°$$

26 degrees seems way too small an angle – and it is. There is a flaw in the sine rule based around the fact that:

$$\sin(180° - \theta) = \sin\theta$$

Whenever you get an angle using the sine rule, you must check to see whether it makes physical sense – if it doesn't then you must use the above rule to find the other possible answer:

$$\sin(180° - 26°) = \sin 26°$$

$$\text{So } \sin 154° = \sin 26°$$

An angle of 154 degrees is much more reasonable for angle B. It also means the sum of angles inside the triangle is 180°, so it looks like the correct answer.